Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani
Home Work #1, Fall 2022

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Sep. 5^{th}, 2022 **Due date:** Sep. 18^{th}, 2022, 11:59 pm

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PROBLEM 1: Consider a discrete memoryless binary *asymmetric* channel with the following conditional error probabilities i.e., $P(r = 1|s = 0) = p$ and $P(r = 0|s = 1) = q$. The random variables r and s stand for receiver and sender. (a) Obtain a closed form expression for the channel capacity. (b) Obtain the error probability as a function of the channel parameters using a three-bit majority voting rule under the mapping $0 \to 000$, $1 \to 111$. How does the error exponent scale with serial concatenations of the code? (15 pts.)

PROBLEM 2: Solve the following exercise problems in Mark Wilde's book: 3.5.4, 3.5.5, 3.5.6, 3.5.13. (10 pts.)

PROBLEM 3: Consider the Klauder-Glauber-Sudarshan states also called *coherent* states, given by $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$, $\alpha \in \mathbb{C}$. $|n\rangle$ represents the number counting states of the Hamiltonian $H = \hbar\omega(\hat{a}^\dagger \hat{a} + \frac{1}{2})$. Here, $\hat{a}^\dagger$ and $\hat{a}$ represent the photon creation and annihilation operators with the property that $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

(1) Prove that the states form a complete basis i.e., $\sum_{n=0}^{\infty} |n\rangle\langle n| = \mathbb{I}$.
(2) Are the states orthogonal? Justify.
(3) Using the Dirac notation learnt in the class, evaluate the average and variance of the photon number in the coherent state.
(4) Do these states satisfy the uncertainty principle? Justify with all the details from first principles.

(25 pts.)

PROBLEM 4: Prove the following results:

(1) Quantum no-deletion theorem. We discussed the theorem statement in the class.
(2) Suppose parties A and B shared an ebit $|\Phi^+\rangle^{AB}$ with no communication between them. Show that if a universal quantum cloner existed, then A could signal to B at a speed greater than the speed of light by exploiting the ebit.

(15 pts.)

PROBLEM 5: A source emits bits in such a way that there are no two consecutive ones occurring anywhere within the sequence. What can you comment on the size of the *typical set* as function of n realizations of such a random sequence? Empirically generate such random sequences from your source model. Plot the sample entropy as a function of n. What do you observe for large n? (10 pts.)