Indian Institute of Science

E9: 252 Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani Mid Term Exam, Spring 2022

Name and SR.No:

Instructions:

- This is an open book, open notes exam.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- There is absolutely no collaboration with any human or a bot.
- This is a take home exam. Assigned on March 26th, 2022, 11:59 pm. Turn in by March 28th, 2022 11:59 pm.
- Do not panic, do not cheat, good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: A signal x[n] with a sampling rate of 20 KHz is to be downsampled by a factor M=50 to produce a 400 Hz signal. The passband of the signal is from 0-150 Hz. The band extending from 150-200 Hz is the transition band. The passband and stop band ripples of 1e-3 are desired. Design the best sampling rate conversion system for this specification. Your solution must be the optimal one in terms of computational complexity. Justify all the details of your work. (35 pts.)

PROBLEM 2: A signal sampled at 50 Mbps must be upsampled to 70 Mbps followed by filtering through a filter H(z). How do you accomplish this through an efficient architecture? Suppose the input signal has a spectrum $X(e^{j\omega})$, what is the spectrum at the output of the convertor? Show all the details of your work carefully. (20 pts.)

PROBLEM 3: Consider a biased Bernoulli source emitting a sequence of binary digits ± 1 . The probability of digit '1' is p. This discrete time sequence is filtered through (a) an IIR filter $H(z) = \frac{a}{1+bz^{-1}}$, and (b) an FIR filter $H(z) = a + bz^{-1}$. Compute the power spectral density at the output of the filters. Suppose a and b take values within [-1,1], comment on the nature of the power spectral density i.e., entirely low pass or high pass etc. (15 pts.)

PROBLEM 4: This problem has four parts:

- (1) Gaussian random samples $\mathcal{N}(0, \sigma^2)$ are quantized through a ceil operator. Compute the sample mean post quantization. Verify your analysis via simulations.
- (2) Is it possible that the autocorrelation matrix of a certain random process is not symmetric? If so,
- justify the nature of the random process through an example. (5 pts.) (3) Obtain a state space representation for $H(z) = \frac{1+z^{-1}}{1+0.5z^{-1}+2z^{-2}}$. Show all your steps analytically. (15
- (4) A student filtered a random discrete time sequence from a source through an unknown LTI filter and analyzed its spectrum. He always observed a dc-component at the output in several runs of his experiments. What inferences can be drawn on the spectra of the source and the spectrum of the filter? (5 pts.)