Indian Institute of Science

Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani
Home Work #1, Spring 2022

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Feb. 8^{th}, 2022 **Due date:** Feb. 21^{st}, 2022, 11:59 pm.

PROBLEM 1: **Basics of probability and random processes**

(1) Consider all points uniformly and jointly distributed within a regular tetrahedron with coinitial edge lengths a. You may assume that the apex aligns with one of the coordinate axes. Obtain the marginal densities and the covariance matrix analytically. Are the random variables statistically independent? Justify. Write a software code to verify your analytical results through alternative means and validate. (15 pts.)

(2) Construct an example of 2-D discrete time random process which is ergodic in the mean. (10 pts.)

(3) Construct an example of a non-Borel set in $\mathbb{R}$ and explain your reasoning. You are allowed to refer to any book or material for this. Try to be original in your approach and come up with your own example. (10 pts.)

(4) A lizard and a cockroach are located on the opposite corners of a regular cube. At each time instant they are allowed to move only along the edges of the cube. Each movement along an edge is equally likely. Calculate the mean time when they meet. This is for extra credit. (15 pts.)

PROBLEM 2: **Basics of linear algebra**

(1) Two matrices A and B that commute with each other have the same eigen bases. Justify if the statement and its converse are true or false. (10 pts.)

(2) Consider the signal set $\left\{ s_i(t) = t^{\frac{1}{i+1}} \right\}_{i=1}^{4}$ defined over the interval $[-1, 1]$. Represent these signals within a signal geometric framework. Is there any approximation error? Let us assume that these signals are transmitted over a channel. Suppose zero mean Gaussian noise with variance σ^2 acts independently on the individual signal coordinates at the receiver. Determine the optimal decision boundaries for classification of these signals and the probability of misclassification. You can assume all the signals are equally likely to be transmitted. (20 pts.)

(3) All square integrable functions are absolutely integrable. Justify if the statement is true or false. (10 pts.)

PROBLEM 3: Consider a serial cascade of N linear and time-invariant systems, where each system S_i, $1 \leq i \leq N$ has a state variable representation $(\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i^T, d_i)$. What is the overall state variable representation of the cascaded system? Work out the details for a parallel cascade. (15 pts.)

PROBLEM 4: An analog signal is sampled at 8Kb/s and reconstructed perfectly using an ideal brickwall filter whose digital cutoff frequency is $\frac{\pi}{16}$.

(1) What can you say about the maximum frequency content in the original analog signal?
(2) Write an expression for the reconstructed analog signal from the sampled values.

(10 pts.)