Indian Institute of Science

Neural Networks and Learning Systems-I

Instructor: Shayan Srinivasa Garani
Home Work #1, Fall 2021

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Sep. 4^{th}, 2021 **Due date:** Sep. 17^{th}, 2021, 11:59 pm

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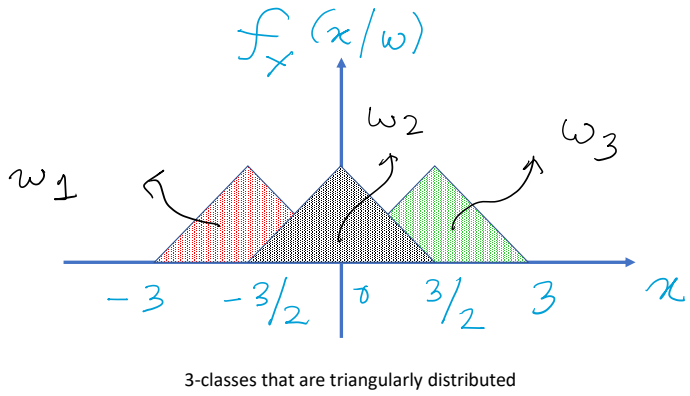
PROBLEM 1: (a) How would you separate all integers modulo N on a real line using linear decision boundaries? (b) Explain in your own words (no more than 50 words) the stochastic resonance effect in neurons. (5 pts.)

PROBLEM 2: Sketch the architecture of a single hidden layer recurrent network with 2 input nodes, 2 hidden nodes and an output node. Self loops and lateral connections are not allowed. Assuming the stochastic neuron model, write down the equations for the signals at the output of each neuron. Indicate all the necessary variables carefully. (3 pts.)

PROBLEM 3: The output response of a certain device obeys the law $y = \frac{x}{a+bx}$, where a, b are positive constants. You make measurements (x_i, y_i) over N data points that could be potentially noisy. Nothing is known about the statistics of the noise. You are required to fit the parameters a and b empirically from the data. How would you accomplish this? Provide explicit expressions for your estimates of a and b using techniques learnt in the class. (7 pts.)

PROBLEM 4:



3-classes that are triangularly distributed

FIGURE 1. 3-classes with points distributed uniformly within triangular regions.

Consider a 3-class classification problem, comprising labels ω_i, $i = 1, 2, 3$ corresponding to data points distributed as shown in Figure 1 supported over $[-3, 0]$, $[-1.5, 1.5]$ and $[0, 3]$ respectively. The corresponding *apriori* probabilities for the classes are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$. Are the points linearly separable? Determine the optimum thresholds and provide a Bayes decision rule to decide the label for a point randomly sampled

from the interval $[-3, 3]$. Compute the probability of misclassification error.

(10 pts.)

PROBLEM 5: Consider the Iris data set https://archive.ics.uci.edu/ml/datasets/iris. We are interested in constructing linear classifiers for this data based on the perceptron.

(1) Identify the classes that are linearly separable. From first principles, write a software code to configure and run the perceptron algorithm in online and batch modes. Provide a plot of the error trajectory as a function of iteration steps/epochs for both online and batch modes. Sketch the final decision boundaries after convergence.
(2) Experiment your results by shuffling the data points every epoch with different learning rates and random weight initializations. What are your conclusions? You may want to make a movie to demo your results using Matlab, Python or other software tools.

(25 pts.)

EXTRA CREDIT: The error trajectory from batch perceptron mode is less noisy than the online mode in the average sense. Justify if the statement is true or false mathematically. (3 pts.)