Indian Institute of Science

Neural Networks and Learning Systems-I

Instructor: Shayan Srinivasa Garani
Homework #0, Fall 2021

This is a self-assessment homework

Assigned date: Aug. 9^{th} 2021. You should not spend more than 3 days on this and no collaboration.

PROBLEM 1: Calculate $\frac{dz}{dt}$ for each of the following functions:

(1) $z = f(x, y) = 4x^2 + 3y^2, x = x(t) = \sin(t), y = y(t) = \cos(t)$
(2) $z = f(x, y) = \sqrt{x^2 - y^2}, x = x(t) = e^{2t}, y = y(t) = e^{-t}$
(3) $z = f(x, y) = x^2 - 3xy + 2y^2, x = x(t) = 3\sin(2t), y = y(t) = 4\cos(2t)$

PROBLEM 2: You need to separate the set of odd and even integers on the real line. Is it possible through a single straight line? Can you come up with an equation that separates the even and odd integers without transforming the points? You need not use any learning technique. Use your common sense.

PROBLEM 3: Write software code to accomplish the following:

(1) Find the second largest element of an array of N numbers.
(2) Generate N points constrained to lie *strictly* inside a circular disk with center at the origin, inner radius a and outer radius b. N can be arbitrary. Plot the points.

PROBLEM 4: Vector Algebra

(1) Find the eigenvalues and eigenvectors of matrix A: $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$
(2) When $a + b = c + d$ show that (1, 1) is an eigenvector and find both eigenvalues : $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
(3) The block B has eigenvalues (1, 2) and C has eigenvalues (3, 4) and D has eigenvalues (5, 7). Find the eigenvalues of the 4 by 4 matrix A:

$$A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

PROBLEM 5: Answer the following:

(1) You are given a discrete time sequence $y[n]$. What is the z-transform of $y[n - k]$?
(2) Solve the equation $y[n] = y[n - 1] + y[n - 2]$ explicitly in closed form. Use the initial conditions $y[0] = 0, y[1] = 1$.

PROBLEM 6: You take a stick of length L and break it into 3 parts, not necessarily of the same length. What is the probability that the broken parts form a triangle? This requires you to think slightly.

PROBLEM 7: Compute the mean and variance of (a) binomial random variable with the usual parameters (b) uniform random variable over $[-a, a]$. Sketch the prob. mass/density function and prob. distribution functions for the cases (a) and (b).

PROBLEM 8: Counting and Probability

(1) There are 3 arrangements of the word DAD, namely DAD, ADD, and DDA. How many arrangements are there of the word PROBABILITY?
(2) Suppose you pick two cards from a deck of 52 playing cards. What is the probability that they are both queens?

(3) Let C and D be two events with $P(C) = 0.25, P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$?
(4) Suppose you want to divide a 52 card deck into four hands with 13 cards each. What is the probability that each hand has a king?

PROBLEM 9: It is your turn to cook a 5 course vegetarian meal for a set of 4 people in your home in 1 hour using 4 stoves within the shortest possible time and minimization of gas/electricity and rush to your office meeting. The menu includes 2 chapatis/person, curry, rice, sambar and chutney. As an engineer/scientist, come up with an algorithm to efficiently accomplish this so that you are not late to attend the meeting, requiring your presence. You can assume that all the ingredients and cutlery/appliances are available to you, and there are no *automated* chapati/chutney/sambar makers. Assume that you know how to make chapatis, curry, chutney and sambar. You are also not allowed to delegate your work to anyone. If unfamiliar with cooking, you can ask your folks at home how to cook during the lock down period. What concepts in computing did you use?

NOTE:

You must be familiar with the basics of linear algebra and probability/statistics, differential equations and other topics at the level of undergrad engg. mathematics. In addition if you do not have basic signals and systems background, go through my MMTSP lecture videos 1-30 on NPTEL site or YouTube. You may not need all of them, but certainly some lectures that cover the basics of linear algebra, probability and signals/systems as required to abstract signals from sensing to data points that we deal with in this course. This is typically the background material I cover in about a month before I begin the actual MMTSP course. Here is a self-scoring chart if you desire to credit NNLS-I.

(1) All problems were solved without referring to any book or notes. You are very well-prepared to take the course and excel.
(2) All problems were solved, but you needed to refer to books/notes sometimes to brush up what you forgot. No worries, you can pick up easily along the way with little effort.
(3) If you solved about 5 problems without referring to any book/notes, you are fine to take the course and can pick up along the way all the missing basics with some effort. Go ahead, confidently.
(4) If you solved only about 5 problems correctly, and had to refer to notes/books all along the way, you need to work hard and pick up on all the background topics in parallel since you need to brush up the unfamiliar concepts.
(5) If you solved less than 4 problems correctly even after referring to the concepts/notes etc. and/or unfamiliar with any of the terms in the assignment, I suggest you to take this course next year after you get the basics strengthened. Do not credit it.

The above cases are suggestive, but not comprehensive. If you are motivated to learn and can work hard, you can read up the background topics and strengthen yourself all along the way.