Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Home Work #2, Spring 2020

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Feb. 20^{th}, 2020 **Due date:** Feb. 28^{th}, 2020

NOTE: You may make any assumptions to solve the problems below. Make sure you state them clearly and justify towards the problem solution.

PROBLEM 1: A stick of length L is cut into three parts randomly. What is the probability that the three cut parts form a triangle? (4 pts.)

PROBLEM 2: Consider a random process $Y(t) = A\sin(\omega t)$. Let A be an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. Sketch the sample functions and the probability density and distribution functions for times $t = 0, \frac{\pi}{3\omega}, \frac{\pi}{2\omega}$.

(6 pts.)

PROBLEM 3: Suppose we are filtering a Bernoulli process with $\Pr(x[n] = 1) = p$ and $\Pr(x[n] = 0) = 1-p$ through the IIR filter $H(z) = \frac{1}{1-az^{-1}}$, $|a| < 1$. Is the output process wide sense stationary? Is it ergodic in the mean? (6 pts.)

PROBLEM 4: Let $f(t) = \sum_{i=0}^{N-1} 2^{-i} u(t - \frac{i}{4}T)$. Here, N is a positive integer, T is the signaling interval and $u(.)$ is the usual unit step function. Let the orthonormal basis derived from the signals $\phi_1(t) = u(t) - u(t - \frac{T}{4})$ and $\phi_2(t) = u(t) - 2u(t - \frac{3T}{16}) + u(t - \frac{T}{4})$ be $\psi_1(t)$ and $\psi_2(t)$.

(1) Express $f(t)$ using $\psi_1(t)$ and $\psi_2(t)$. Plot the signal trajectory in an appropriate signal plane for $N = 5$. A Matlab code can help towards computation and the plotting once you set up the equations.

(2) Suppose a communication source emits $\psi_1(t)$ and $\psi_2(t)$ with probabilities p and $1 - p$ respectively. Imagine a 2D Gaussian cloud with mean zero and covariance matrix Σ acting on the points in the signal space. Derive the conditions for the optimal decision boundary to reduce the probability of misclassification.

(14 pts.)