Indian Institute of Science

E9-207: Basics of Signal Processing

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Mid Term Exam#2, Spring 2018

Name and SR.No:

Instructions:

- You are allowed only 4 pages of written notes and a calculator for this exam. No wireless allowed.
- The time duration is 3 hrs.
- There are six main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
6	
Bonus	
Total points	

PROBLEM 1: Simplify the following multirate systems shown in Figure 1 as best as you can. Obtain the simplified frequency response. Show all your steps carefully. (15 pts.)

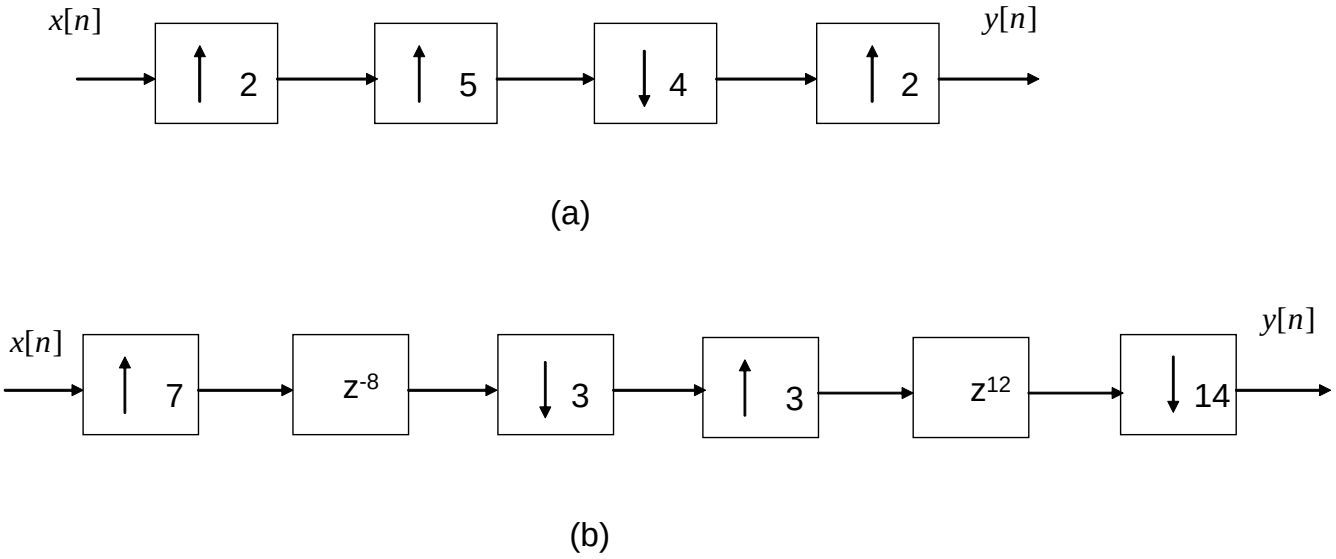

FIGURE 1. Multirate systems.

PROBLEM 2: Prove that decimation by M followed by expansion by L can be interchanged if L and M are relatively prime. You must prove this result in the frequency domain representation. (10 pts.)

PROBLEM 3: This problem has two parts

- A student was performing measurements on an oscillator. Over what period must the signal be averaged so that he can claim that the device was producing frequencies accurately up to 0.25 KHz? (5 pts.)
- Suppose we are filtering a natural image using (a) 2D FFT (b) 2D Haar wavelet. Sketch the frequency resolution in the 2D $\omega_1 - \omega_2$ frequency plane for both the cases. You can assume a N-level dyadic decomposition. (15 pts.)

PROBLEM 4: Consider a 3-channel filterbank with analysis filters $H_0(z)$, $H_1(z)$ and $H_2(z)$ across first three branches respectively. Consider decimation rates over the three branches to be (a) (2,3,6) (b) (2,4,4). Derive the conditions for alias free reconstruction from first principles. If the synthesis filters are $F_0(z)$, $F_1(z)$ and $F_2(z)$, obtain the alias free distortion function. (25 pts.)

PROBLEM 5: Let $H_0(z) = 1 + 2z^{-1} + 4z^{-2} + 2z^{-3} + z^{-4}$. The analysis filters are quadrature mirror symmetric. Draw an implementation for the pair $[H_0(z)\ H_1(z)]$ in the form of a uniform DFT analysis bank, explicitly showing the polyphase components, the 2×2 IDFT box and relevant details. (20 pts.)

PROBLEM 6: Expand the signal $s(t) = t^3$ in the interval [0,1] using Haar wavelets up to a resolution of 0.25. (10 pts.)

BONUS POINTS: Examine if the function $f(t) = t^2$ is uniformly continuous over $(0, \infty)$ (10 pts.)