Indian Institute of Science

E9−207: Basic of Signal Processing

Instructor: Shayan G. Srinivasa

Homework #3, Spring 2018

Late submission policy: Points scored = Correct points scored × e^{-d}, d = # days late

Assigned date: Mar. 15th 2018 **Due date**: Mar. 27th 2018 in class

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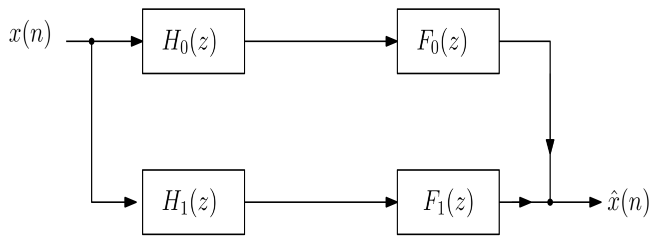
PROBLEM 1:

(a)Let $H(z) = \frac{1+az^{-1}}{a+z^{-1}}$, $a \in \mathbb{R}$. Write down the expressions for the Type 1 polyphase components (with $M = 2$). What can you say about $H(z)$ for various values of a? (4 points)

(b)Let $H(z) = \frac{1}{1-2R\sin\theta z^{-1}+R^2z^{-2}}$ with $R > 0$ and $\theta \in \mathbb{R}$. Find the Type 1 polyphase components for $M = 2$. (4 points)

PROBLEM 2:

Consider the analysis/synthesis system shown below:



(a) Let the analysis filters be $H_0(z) = 1 + 3z^{-1} + \frac{1}{3}z^{-2} + z^{-3}$ and $H_1(z) = H_0(-z)$. Find causal stable IIR filters $F_0(z)$ and $F_1(z)$ such that $\hat{x}(n)$ agrees with $x(n)$ except for a possible delay and (non zero) scale factor.(5 points)

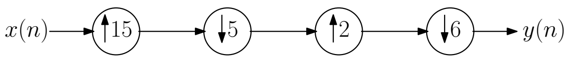
(b) Let the analysis filters be $H_0(z) = 1 + z^{-1} + 3z^{-2} + z^{-3} + z^{-4}$ and $H_1(z) = H_0(-z)$. Find causal stable FIR filters $F_0(z)$ and $F_1(z)$ such that $\hat{x}(n)$ agrees with $x(n)$ except for a possible delay and (non zero) scale factor. (5 points)

PROBLEM 3:

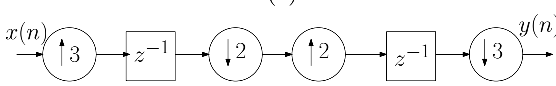
Let $H_0(z) = \frac{1+2z^{-1}}{2}$. Find the real coefficient causal FIR filter $H_1(z)$ such that the pair $(H_0(z), H_1(z))$ is power complementary. Are these filters also all pass complementary? (6 points)

PROBLEM 4:

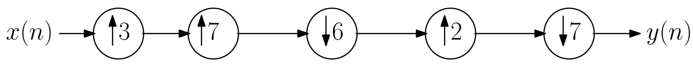
Simplify the following multirate systems shown below as best as you can. Obtain the z-transform of the output signal in terms of that of the input signal. (3 × 4 = 12 points)



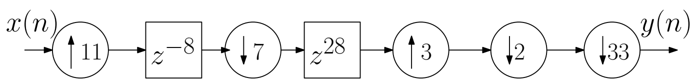
(a)



(b)



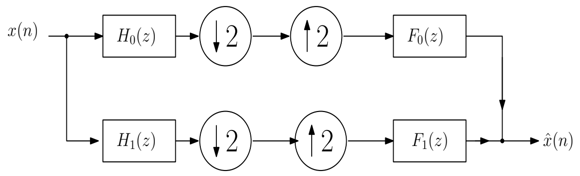
(c)



(d)

PROBLEM 5:
Prove that decimation by M followed by expansion by L can be interchanged if L and M are relatively prime. You must prove this result in the time and frequency domain representations. (10 points)

PROBLEM 6:
Consider the two channel QMF bank shown below where the analysis filters are given by



$$H_0(z) = 2 + 6z^{-1} + z^{-2} + 5z^{-3} + z^{-5}; H_1(z) = H_0(-z).$$

Find a set of stable synthesis filters that result in perfect reconstruction. (4 points)