Indian Institute of Science
E9−207: Basic of Signal Processing
Instructor: Shayan G. Srinivasa
Homework #2, Spring 2018
Late submission policy: Points scored = Correct points scored × e^{-d}, d = # days late
Assigned date: Feb. 20$^{\text{th}}$ 2018 **Due date**: Mar. 1$^{\text{st}}$ 2017 in class

---

PROBLEM 1:
(a) Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two vector spaces. Then show that $\mathcal{S}_1 \cap \mathcal{S}_2$ is also a vector space. (3 pts)
(b) If A $\in \mathbb{C}^{n\times n}$ and $u, v \in \mathbb{C}^n$ are non-zero vectors such that $Au = 2u$ and $Av = 3v$, show that u, v are linearly independent. (5 pts)
PROBLEM 2:
(a) Let $A \in \mathbb{C}^{m\times m}$ be a matrix acting on vectors in the vector space $\mathbb{C}^m$. We define a new product between vectors $x, y \in \mathbb{C}^m$ as $\langle x, y\rangle_A$ as $x^\dagger Ay$. Under what conditions on A is this a valid inner product? (5 pts)
(b) Consider the matrix

$$A = \begin{pmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{pmatrix}$$

For what values of $a \in \mathbb{C}$ is $\sqrt{x^\dagger Ax}$ a norm defined on $\mathbb{C}^3$? (5 pts)
Note: $a^\dagger$ is the transpose conjugate of a. For example

$$v = \begin{pmatrix} 1 \\ i \\ -i \end{pmatrix} \Rightarrow v^\dagger = \begin{pmatrix} 1 & -i & i \end{pmatrix}$$

PROBLEM 3:
What is the minimum value of $x - y - z$ subject to the constraint $x^2 + y^2 + z^2 = 1$? (4 pts)
PROBLEM 4:
Consider the functions $\varphi_k(t) = A\,\mathrm{sinc}(\pi(t-k))$ where k is an integer and $A \in \mathbb{C}$.
For integers k, l evaluate

$$\int_R \varphi_k(t)\varphi_l^*(t)dt$$

Conclude that $\varphi(t) \in L^2(\mathbb{R})$ and that $\{\varphi_k : k \in \mathbb{Z}\}$ forms an orthonormal set of functions in $L^2(\mathbb{R})$. (8 pts)
PROBLEM 5:
(a)A baseband signal $s(t)$ with 50 Hz bandwidth is sampled at a rate F_s. The resultant signal is downsampled by a factor 2 to obtain the discrete samples $\hat{s}(n)$. What is the minimum value of F_s in Hz to reconstruct back the signal $s(t)$ from the samples $\hat{s}(n)$? (4 pts)
(b) Let $s(n)$ be any discrete time signal with energy E_s. The signal is downsampled by 3. What is the evergy of the resultant signal if there is no aliasing after decimation? (4 pts)
PROBLEM 6:
(a) A signal $x(t)$ is obtained by convolving signals $x_1(t)$ and $x_2(t)$ with the following characteristics:

$$|X_1(\omega)| = 0 \text{ for } |\omega| > 500\pi,$$
$$|X_2(\omega)| = 0 \text{ for } |\omega| > 250\pi.$$

Impulse train sampling is performed on $x(t)$ to get $x_s(t) = \sum_{-\infty}^{\infty} x(nT)\delta(t - nT)$. Specify the range of values of T so that $x(t)$ may be recovered from $x_s(t)$. (4 pts)
(b) The signal $s(t) = \begin{cases} 1 - |t| \text{ for } -1 \le t \le 1 \\ 0 \text{ otherwise.} \end{cases}$ is passed through a system to obtain the output $\hat{s}(t)$. The system has a resonant frequency of $\frac{2}{3}$ Hz and hence allows only frequencies of $\frac{2}{3}$ Hz and its harmonics along with d.c. component. What is the value of $\int_{-2}^{2} |\hat{s}(t)|^2$? (8 pts)