Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani

Mid Term Exam#2, Fall 2017

Name and SR.No:

Instructions:

- You are allowed only 5 pages of written notes and a calculator for this exam. No wireless allowed.
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
Total points	

PROBLEM 1: An analog signal $s(t)$ has a spectrum as shown in Figure 1. The maximum frequency in the signal as per the spectrum is $\omega_{\max}$ Hz. According to the sampling theorem, we must be able to reconstruct the signal by sampling at a rate $f_s > 2\omega_{\max}$ followed by an anti-aliasing filter. Is it possible to sample at less than the Nyquist rate and reconstruct the signal? Discuss the situation carefully. (15 pts.)
Hint: You do not need any compressive sampling or other recent sampling ideas except what has been discussed in the class.

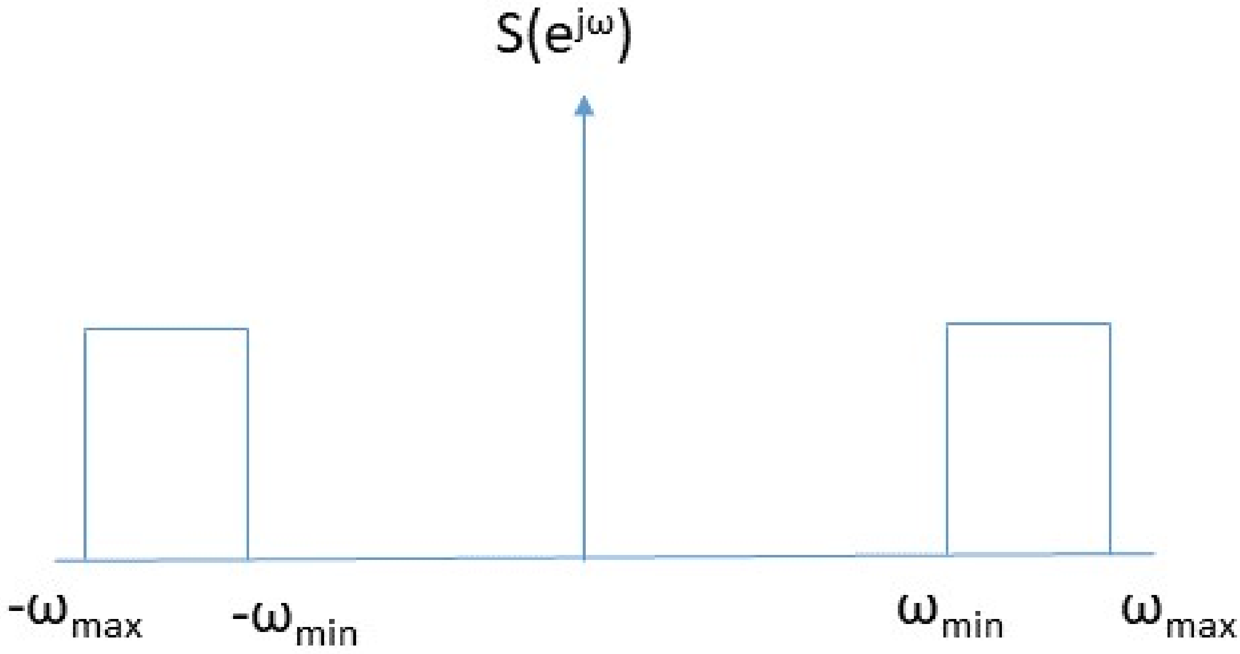

FIGURE 1. Spectrum of an analog signal.

PROBLEM 2: A student was performing measurements on a frequency reporting device. In an experimental report, the student mentioned that, while experimenting with pure sine waves, to account for frequency deviations of the device, he averaged the signal over a 0.5 ms time interval. He claimed that the device was producing frequencies accurately up to 0.1 KHz. Are the conclusions correct? Justify. (10 pts.)

PROBLEM 3: Let $\mathcal{V}_p$ denote the space comprised of all functions $s(t) \in L^2(R)$ such that the support of $s(t)$ is over $[-2^{-p}, 2^{-p}]$. Examine if $\{\mathcal{V}_p\}_{p\in\mathcal{Z}}$ is a multiresolution analysis. You need to examine all the properties. (25 pts.)

PROBLEM 4: Expand the signal $s(t) = t^2$ over the interval [-1,1] using Haar wavelets up to a resolution of 0.25. Sketch the wavelet expansion. What is the approximation error? (20 pts.)

PROBLEM 5: You are given a digital low pass FIR filter $H(z)$ of order N and no other filter. You are also given a sampled speech signal $x[n]$ from an analog signal of duration 2 minutes sampled at 16KHz using a 12 bit ADC. You are required to realize a speech compression engine.

(1) What is the original bit rate and the signal length in bits? (5 pts.)

(2) Suppose we decide to have J stages over dyadic subband coding as shown in Figure 2(A). Show the equivalent analysis bank of filters i.e., just an equivalent filter and a downsampler on each branch as in Figure 2(B) . What is the bit rate without any compression at each branch? You can assume that linear convolution is done. (7 pts.)

(3) Suppose we need a target compression rate of 1:R, propose a scheme as to how you would reduce the bit rate using just quantizers. You may use a uniform or non-uniform quantizer as appropriate. Justify your choice. (10 pts.)

(4) Sketch the synthesis stage, and show all the equivalent synthesis filters in each branch along with the necessary expanders similar to sub-part (2) above. (8 pts.)

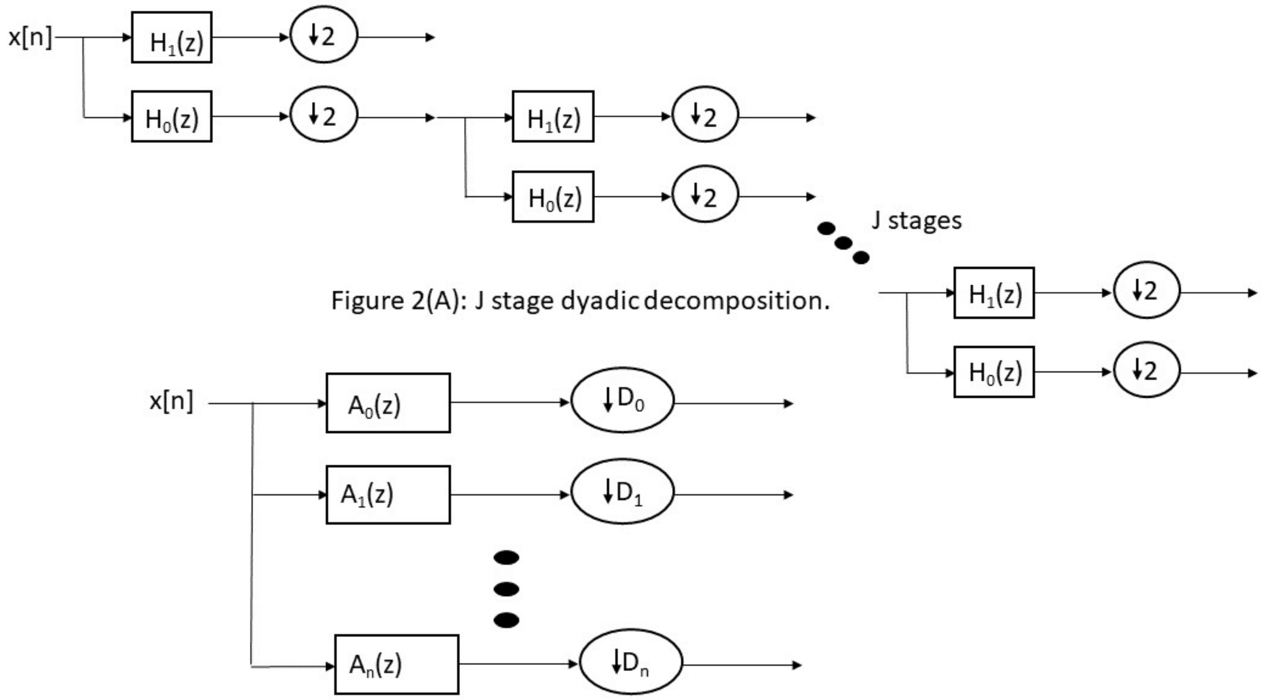

Figure 2(B): Equivalent representation of Figure 2(A).

FIGURE 2. Subband coding of speech.