Indian Institute of Science

CCE: Neural Networks for Signal Processing -I

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Mid Term Exam#1, Spring 2017

Name and SR.No:

Instructions:

- This is a take home exam. You must turn in your solutions neatly written by March 6^{th} 2017 before the day ends.
- There are three main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- There is absolutely no collaboration with any one except referring to any books and class notes. You must cite the references sources if any.
- Do not panic, do not cheat, good luck!

Question No.	Points scored
1	
2	
3	
Total points	

PROBLEM 1: This problem has 3 parts.

(1) Construct a recurrent neural network with 3 input, 3 hidden and 3 output neurons. Write down the equations at each stage of the network. Assume that self loops are avoided. (5 pts.)

(2) Consider an autoregressive process given by $y[n] = \sum_{i=1}^{M} w_i y[n-i] + v[n]$ where $v[n]$ is a zero mean Gaussian noise with variance σ^2. Here, M is the memory order. Is this process scale and time translation invariant? Justify. How could these invariances be used in neural networks? (5 pts.)

(3) Consider a multilayer neural network where neurons operate in the linear region. This network is equivalent to a single layer feed forward network. Justify if the statement is true or false. (10 pts.)

PROBLEM 2: This problem has 2 parts.

(1) Consider two 1D Gaussian distributions with densities $\mathcal{N}(3, 10)$ and $\mathcal{N}(-3, 5)$. Assuming a set of samples are drawn from these distributions with equal apriori probabilities. Are they linearly separable. Design a classifier for separating them. Show all your steps carefully. (10 pts.)

(2) Consider the cost function for a perceptron based on misclassified samples. $J_p(\mathbf{w}) = \sum_{\mathcal{S}(\mathbf{w})} -(\mathbf{w}^T\mathbf{x})$, where $\mathcal{S}(\mathbf{w})$ is the set of samples misclassified by the perceptron for a choice of the weight vector $\mathbf{w}$. The outputs are misclassified if $\mathbf{w}^T\mathbf{x} \leq 0$. Interpret the cost function $J_p(\mathbf{w})$ geometrically. Using gradient descent, derive an update rule for adjusting the weights for this perceptron. Prove that your result converges mathematically. (25 pts.)

PROBLEM 3: Computer Experiments. This problem has 2 parts.

(1) Let us consider a simplified version of a recognition system. Consider two geometric shapes i.e., a rhombhus and an equilateral triangle whose centroids are at the origin. It is desired to develop an algorithm that recognizes these shapes irrespective of their rotations in discrete steps of an angle θ. From first principles, develop the procedure using the back propagation algorithm using a sigmoid activation function. Demonstrate the working of your architecture by generating these shapes and feeding it to your neural network. You must provide the original source code, discuss the influence of the learning rate, choice of initial weights, maximum iteration steps and other configuration parameters in full details. Show all your work carefully. (30 pts.)

(2) It is desired to learn the function $f(x) = \frac{1}{\sqrt{1+x}}$, $1 \leq x \leq 5$. Generate the samples from this target function. Show how the mapping is accomplished using the back propagation algorithm using sigmoid activations with a single hidden layer. You must comment on the network configuration parameters as you experiment with test samples. (15 pts.)