Indian Institute of Science

E9−252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Homework #3 Solutions, Fall 2017

Solutions prepared by Priya J Nadkarni

Late submission policy: Points scored = Correct points scored $\times\ e^{-d}$, $d = \#$ days late

Assigned date: Sept. 11th 2017 **Due date**: Sept. 18th 2017 by end of the day

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PROBLEM 1:

Solve problems 4.6 and 4.11 from P. P. Vaidyanathan's book. (8 + 10 points)

Solution:

(4.6) For the first system, we obtain,

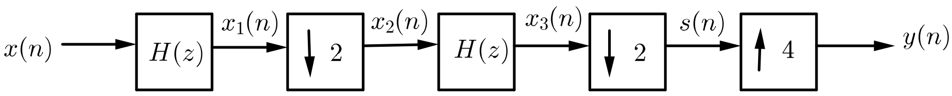
$$
\begin{aligned}
y_0(n) &= h_k(n) * y(n) \Rightarrow Y_0(z) = H_k(z)Y(z) \\
h_k(n) &= h_0(n)\cos\frac{2\pi kn}{L} = \frac{h_0(n)e^{\frac{j2\pi kn}{L}} + h_0(n)e^{\frac{-j2\pi kn}{L}}}{2} \\
H_k(z) &= \frac{H_0(e^{\frac{-j2\pi k}{L}}z) + H_0(e^{\frac{j2\pi k}{L}}z)}{2} \\
Y_0(z) &= \frac{H_0(e^{\frac{-j2\pi k}{L}}z)Y(z) + H_0(e^{\frac{j2\pi k}{L}}z)Y(z)}{2}
\end{aligned}
$$

For the second system, $y_1(n) = (h_0(n) * y(n))\cos\frac{2\pi kn}{L}$. Let $x(n) = h_0(n) * y(n)$, then,

$$
\begin{aligned}
X(z) &= H_0(z)Y(z) \\
Y_1(z) &= \frac{X(e^{\frac{-j2\pi k}{L}}z) + X(e^{\frac{j2\pi k}{L}}z)}{2} \\
&= \frac{H_0(e^{\frac{-j2\pi k}{L}}z)Y(e^{\frac{-j2\pi k}{L}}z) + H_0(e^{\frac{j2\pi k}{L}}z)Y(e^{\frac{j2\pi k}{L}}z)}{2}
\end{aligned}
$$

$Y_0(z) = Y_1(z)$ only if $Y(e^{\frac{-j2\pi k}{L}}z) = Y(z)$. Otherwise, $Y_1(z) \neq Y_0(z)$. One such case where they are equal is when k mod $L = 0$. Another example where it is equal is when the signal has been upsampled by L (Refer Prob 4.5). Thus, they are not necessarily the same. They may be same in some cases, but differ in others.

(4.11) Consider the following system:



When we pass x(n) through the filter $H(z)$, we obtain

$$X_1(z) = X(z)H(z)$$

When we downsample $x_1(n)$ by 2, we obtain,

$$X_2(z) = \frac{X_1(z) + X_1(-z)}{2}$$

When we pass $x_2(n)$ through the filter $H(z)$, we obtain,

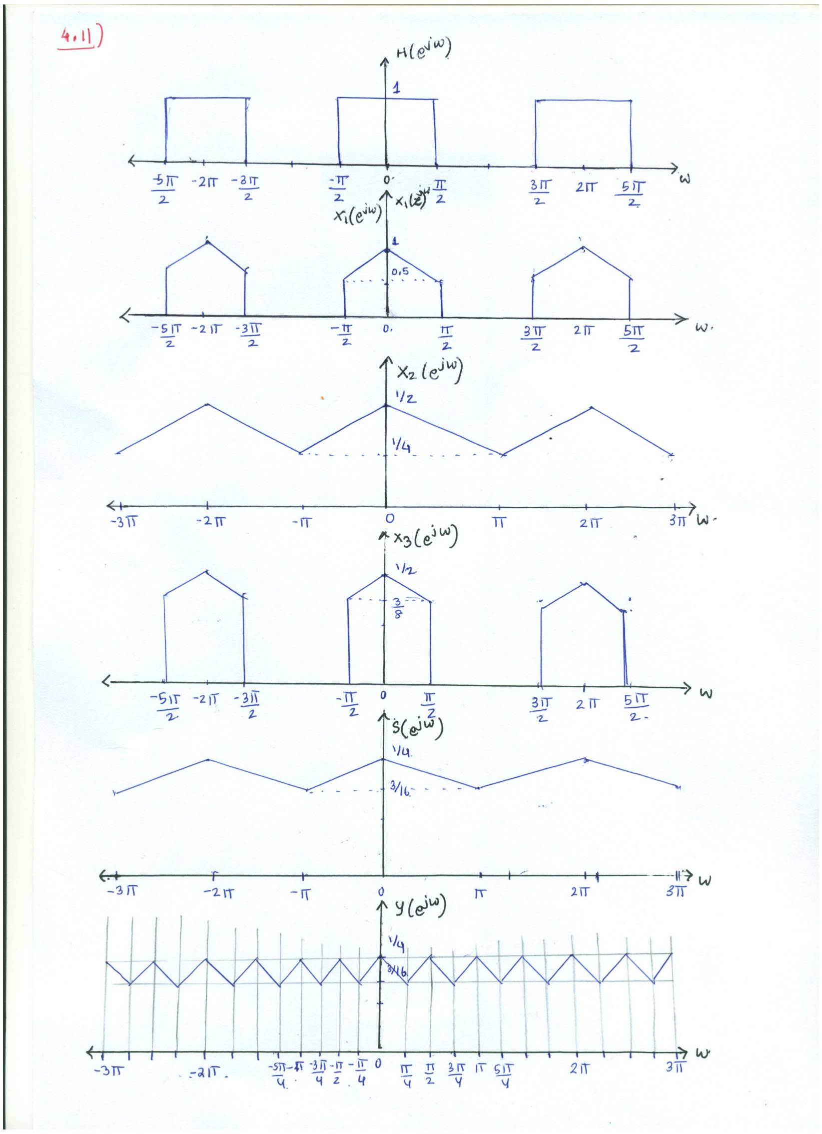
$$X_3(z) = X_2(z)H(z)$$

When we downsample $x_3(n)$ by 2, we obtain,

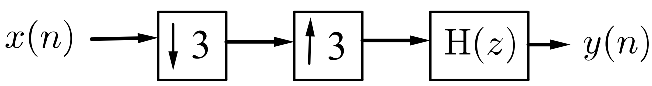
$$S(z) = \frac{X_3(z) + X_3(-z)}{2}$$

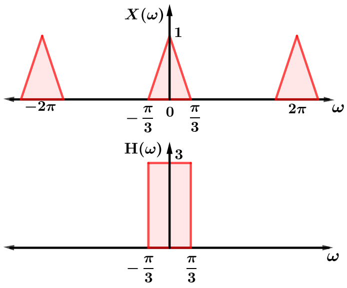
When we upsample $s(n)$ by 4, we obtain,

$$Y(z) = S(z^4)$$



PROBLEM 2:
Consider the following system:

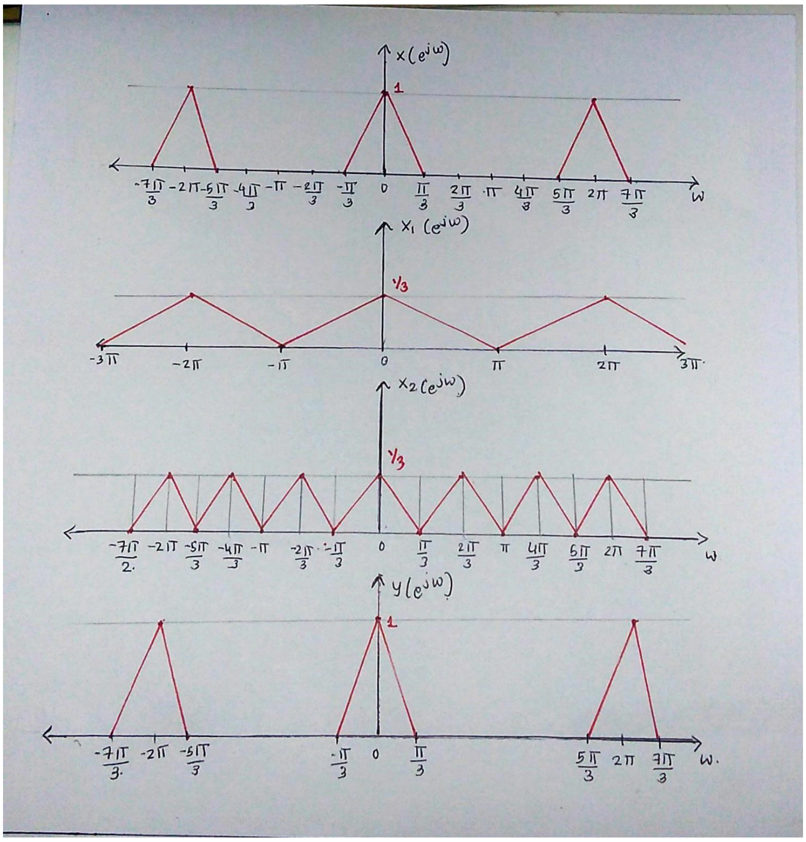




Suppose the spectrum of the original signal and transfer function is as given above.

Analyze the spectrum of $y(t)$. Analyze the output spectrum if the decimator and expander and interchanged. (4+3 points)

Solution: Let $x_1(n)$ and $x_2(n)$ denote the intermediate signals.



If the decimator and the interpolator are exchanged,

$$
\begin{aligned}
X_1(z) &= X(z^3) \\
X_2(z) &= \frac{1}{3}\sum_{k=0}^{2} X_1(z^{1/3}e^{-j\frac{2\pi k}{3}}) \\
&= \frac{1}{3}\sum_{k=0}^{2} X((z^{1/3}e^{-j\frac{2\pi k}{3}})^3) = \frac{1}{3}\sum_{k=0}^{2} X(z) = X(z) \\
Y(z) &= 3X(z)
\end{aligned}
$$

The last equality follows as $X(z)$ is bandlimited to $\frac{\pi}{3}$.