

Indian Institute of Science

E9–252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Homework #1, Fall 2017

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, $d = \#$ days late

Assigned date: Aug. 28th 2017

Due date: Sept. 4th 2017 by end of the day

PROBLEM 1:

Can convolution operator be expressed as an inner product? Justify.

PROBLEM 2:

Define inner products of vectors defined over a complex field \mathbb{C} . For complex vectors x and y , compute $\langle x - y, x - y \rangle$ using the inner product defined. Derive the Cauchy-Schwarz inequality for complex vectors.

PROBLEM 3:

a) Let $S_p = \{x : \|x\|_p \leq 1\}$. Prove that $S_p \subset S_{p+1}$.

b) Prove that $\lim_{p \rightarrow \infty} \mathcal{L}_p = \mathcal{L}_\infty$.

PROBLEM 4: A function $f : X \rightarrow \mathbb{R}$ is called convex if

$$f(\alpha x_1 + (1 - \alpha)x_2) \leq \alpha f(x_1) + (1 - \alpha)f(x_2) \quad \forall x_1, x_2 \in X \text{ and } \alpha \in [0, 1].$$

Examine if $\text{norm}(\cdot)$ is a convex function.