Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Home Work #3, Fall 2015

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Oct. 22^{nd} 2015 **Due date:** Oct. 30^{th} 2015

PROBLEM 1: If $\mathcal{V}$ and $\mathcal{W}$ are finite dimensional orthogonal subspaces of an inner product space $\mathcal{H}$, prove that $\dim(\mathcal{V} \oplus \mathcal{W}) = \dim(\mathcal{V}) + \dim(\mathcal{W})$. (3 pts.)

PROBLEM 2: Obtain the Haar wavelet decomposition of the signal $f(t)$. Indicate the signal dimension at each subspace carefully. Devise a generic algorithm for doing a Haar decomposition using a computer program.

$$f(t) = \begin{cases} 2 & -2 \leq t < -1 \\ -4 & -1 \leq t < -0.5 \\ -2 & -0.5 \leq t < 0 \\ 2 & 0 \leq t < 0.25 \\ 1 & 0.25 \leq t < 2 \end{cases}$$

(12 pts.)

PROBLEM 3: Prove the following properties for Haar wavelets:

- Parseval's equality i.e., energy conservation relation.
- Orthogonality across scales and time translates.

(10 pts.)

PROBLEM 4: For $j \in \mathbb{Z}$, let $\mathcal{V}_j$ be the space of all signals $f(t) \in L^2$ bandlimited within the interval $\left[-2^j\pi, 2^j\pi\right]$. Consider the signal $\phi(t) := \operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$. Prove the following.

- The nesting, closure, shrinking and scaling properties that we discussed in the class as part of the multiresolution analysis definition.
- $\{\phi(t-k), k \in \mathbb{Z}\}$ is a shift orthogonal basis for $\mathcal{V}_0$.
- $\phi(t) = \phi(2t) + \sum\limits_{k \in \mathbb{Z}} \frac{2(-1)^k}{(2k+1)\pi}\phi(2t-2k-1)$. (Scaling relation)

(25 pts.)