Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #2, Fall 2014

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Sep 19^{th} 2014 **Due date:** Sep 26^{th} 2014

PROBLEM 1: Prove that decimation by M followed by expansion by L can be interchanged if L and M are relatively prime. You must prove this result in the time and frequency domain representations. Hint: You must apply the ideas from the lecture notes. (This is adapted from the text by P. P. Vaidyanathan, problems 4.7 and 4.8.) (7 pts.)

PROBLEM 2: Simplify the following multirate systems shown in Figure 1 as best as you can. Obtain the frequency response. (13 pts.)

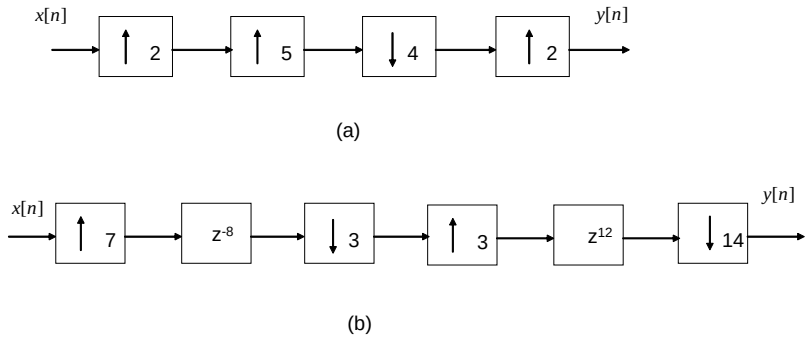

FIGURE 1. Two different multirate systems are shown in (a) and (b).

PROBLEM 3: Suppose we wish to design a 20 fold low-pass digital interpolator. Let the input signal $x[n]$ be bandlimited to $|\omega| < 0.8$. Suppose the ripple specifications for $H(z)$ in the passband and stopband are $\delta_p = 0.01$ and $\delta_s = 0.005$ respectively.

(1) Find the cutoff frequencies ω_p and ω_s for $H(z)$.
(2) What is the filter order for a direct design implementation?
(3) Suppose a two stage design approach is adopted, what are the filters orders? Obtain the frequency responses. Show all the details of your work.
(4) Compare the computational efficiencies of the two stage design and the direct design for a communication signal bandlimited to 3KHz with a 20% oversampling above the Nyquist rate.
(5) Optimize your design over all integer choices of two stage expansion rates. What would you recommend?

(25 pts.)

PROBLEM 4: In the class, during the discussion of rational sampling rate alteration by a factor 1.5, we realized an efficient architecture starting from the interpolation stage. Derive an efficient architecture starting from the decimation filters. Carefully assess if the computational efficiency is the same for both the cases. (5 pts.)