

Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#2, Fall 2014

Name and SR.No:

Instructions:

- This is an open book, open notes exam. No wireless allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: This problem has 2 parts. Let $\phi(t) = \sum_k c_k \phi(2t - k)$ where $\phi(t)$ is the usual Haar scaling function over $[0,1]$, $\{\phi(2t-k)\}$ are an orthonormal basis in \mathcal{V}_{-1} with $\mathcal{V}_0 \subset \mathcal{V}_{-1}$ and c_k are the corresponding Haar coefficients.

(1) Prove that $\sum_k c_k = 2$.

(10 pts.)

(2) Let $\phi(\omega)$ be the continuous time Fourier transform of $\phi(t)$. Prove that $\phi(\omega) = \frac{1}{2} \left(\sum_k c_k e^{-\frac{j\omega k}{2}} \right) \phi\left(\frac{\omega}{2}\right)$.

(10 pts.)

PROBLEM 2: This problem has 2 parts.

(1) Obtain the Haar wavelet decomposition of the signal $s(t) = t^3$ over the interval $[0, 1]$ using the functions $\phi(t)$, $\psi(t)$, $\psi(2t)$ and $\psi(2t - 1)$. The functions $\phi(t)$ and $\psi(t)$ are the usual Haar scaling and wavelet functions over the interval $[0,1]$. (15 pts.)

(2) With the usual notations as followed in the class, prove that $\mathcal{V}_j = \mathcal{V}_J \oplus \bigoplus_{k=0}^{J-j-1} \mathcal{W}_{J-k}$. You must explain all the steps throughout the proof carefully. (15 pts.)

PROBLEM 3: Suppose you are given a color image of size $N \times N$ specified in terms of R, G and B attributes. Note that R, G, B are red, green and blue colors used for describing a pixel within the image as in Figure 1. You can conveniently ignore the intensity attribute throughout this problem. Devise an algorithm for obtaining a *monochromatic* i.e., single color image from the color image using an appropriate linear transformation method. You need to describe the algorithmic steps clearly i.e., specifying the inputs, the procedure and outputs. You must indicate the dimensions of all the vectors and matrices while describing your procedure.

NOTE: You can make reasonable assumptions on any of the variables needed for the algorithm, but must clearly state them. (25 pts.)

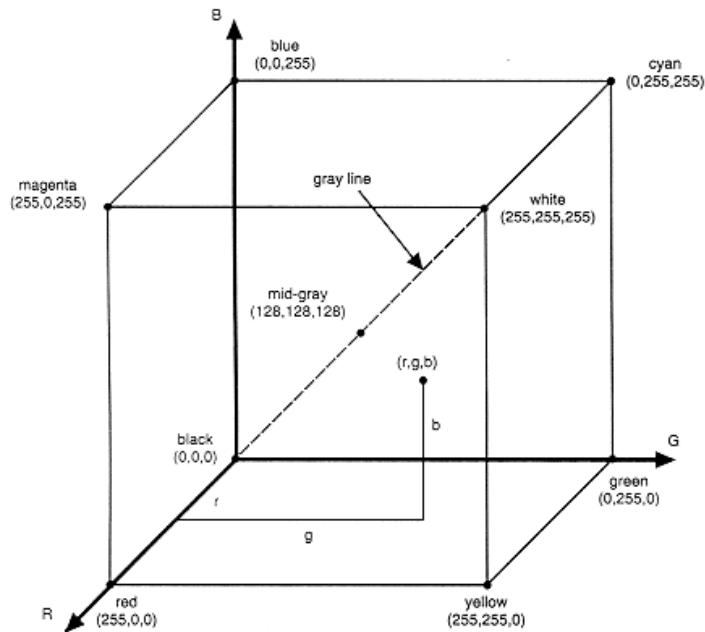


FIGURE 1. Color representation using R, G, B basis.

PROBLEM 4: You are given a Padé model having 3 poles as $H_1(z) = \frac{1}{1+2z^{-1}+z^{-2}+3z^{-3}}$.

- (a) How many input samples does this model exactly match? (2 pts.)
- (b) Determine the input samples that match with the model exactly as in the previous part. (15 pts.)
- (c) What would be the input samples if the Padé model had 3 zeros as $H_2(z) = 1 + 2z^{-1} + z^{-2} + 3z^{-3}$? (5 pts.)
- (d) Suppose ϵ_1 and ϵ_2 are the modeling errors for $H_1(z)$ and $H_2(z)$ for some infinitely long unknown input whose first few samples were to match. Justify if the claim $\epsilon_1 \geq \epsilon_2$ is true/false. A counterexample is enough to make it false. (3 pts.)