Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #4, Fall 2013

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Oct 8^{th} 2013 **Due date:** Oct 18^{th} 2013 in class

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PROBLEM 1: Consider the analysis/synthesis filter bank shown in Figure 1.

(1) Let the analysis filters be $H_0(z) = 1 + 3z^{-1} + 0.5z^{-2} + z^{-3}$ and $H_1(z) = H_0(-z)$. Find causal stable IIR synthesis filters such that $\hat{x}[n]$ agrees with $x[n]$ with a possible delay and scale factor.

(2) Let $H_0(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$ and $H_1(z) = H_0(-z)$. Find causal FIR synthesis filters such that $\hat{x}[n]$ agrees with $x[n]$ with a possible delay and scale factor.

(10 pts.)

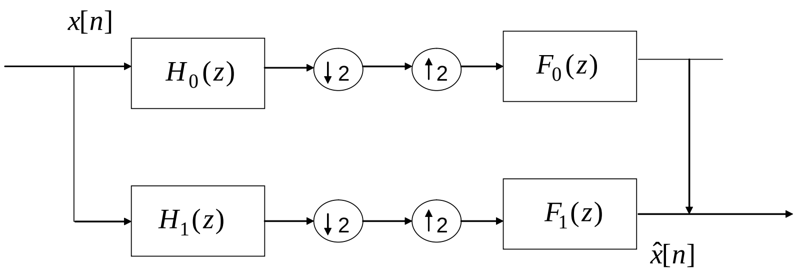

FIGURE 1. Two channel filter bank.

This problem is from the text by P. P. Vaidyanathan problem 4.28.

PROBLEM 2: Consider the following M channel multirate system as shown in Figure 2, which is essentially a QMF bank with additional transfer functions $C_k(z)$ introduced. We can imagine that $C_k(z)$ represents the amplitude and phase distortions introduced by the k^{th} channel.

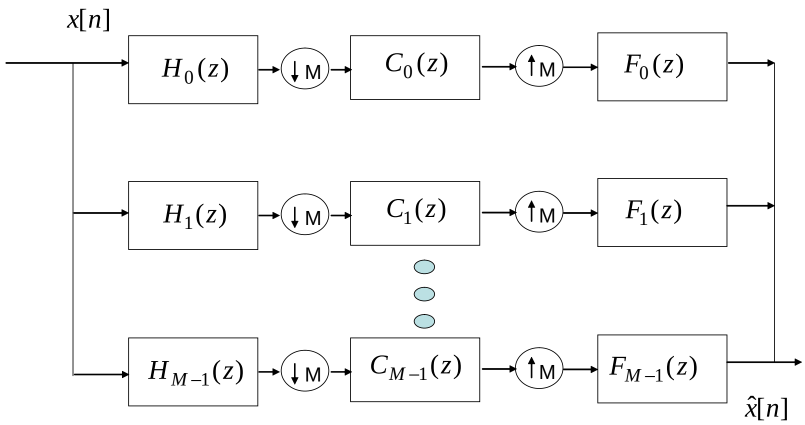

FIGURE 2. M-channel multirate system.

Assume throughout that the functions $H_k(z)$, $F_k(z)$ and $C_k(z)$ are rational and stable unless stated otherwise. Do not make any assumptions about the zeros of these transfer functions.

(1) Let $H_k(z)$ and $F_k(z)$ be such that the system is 'alias free' in the absence of channel distortion $C_k(z)$. Let $C_k(z) \neq 1$. Determine a set of modified stable synthesis filters to retain alias-free property.

(2) Repeat part (a) with the additional constraint 'free from amplitude distortion'. Assume that $C_k(z)$ has no zeros on the unit circle.

(3) Repeat part (a) by replacing 'alias free' with 'perfect reconstruction' everywhere. Assume now that $C_k(z)$ has no zeros on or outside the unit circle.

(15 pts.)

This problem is from the text by P. P. Vaidyanathan problem 5.23.

PROBLEM 3: Suppose we wish to design a 25-fold low-pass linear-phase interpolator. Let the input signal $x[n]$ be bandlimited to $|\omega| < 0.8$. Suppose the ripple specifications for $H(z)$ are $\delta_1 = 0.01$ and $\delta_2 = 0.005$.

(1) Find the cutoff frequencies ω_p and ω_s for $H(z)$.

(2) What is the filter order if a direct design is used?

(3) Suppose a two-stage implementation is used, what are the filter orders? Show all the details of the frequency responses.

(4) Compare the computational efficiencies for the two-stage and direct design if the input signal is a speech signal bandlimited to $4KHz$ under Nyquist sampling rate at the input.

(15 pts.)

PROBLEM 4: Four 2-D data points are given. They are $(3,1)^T$, $(-3,-1)^T$, $(0,5)^T$ and $(-5,1)^T$. The probabilities of these points are $\{0.2, 0.3, 0.3, 0.2\}$ respectively.

(1) What is the KL representation of these points?

(2) Suppose we intend to retain only the dominant eigen component, what is the new representation of these points? What fraction of the signal energy is retained after dimensionality reduction?

(3) How many linear hyperplanes are needed to classify the 4 signal points in 2-D? What would this correspond after dimensionality reduction?

(4) Obtain the probability of error for signal classification using an optimal single hyperplane after dimensionality reduction. Show all your steps and reasoning. (10 pts.)