Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #2, Fall 2013

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Sep 7th 2013

Due date: Sep 17^{th} 2013 in class

PROBLEM 1: Let $X = \cos(\theta)$ and $Y = \sin(\theta)$ denote two random variables, where, $\theta \sim U[0, 2\pi]$. Examine if X and Y are (a) independent (b) correlated. (5 pts.)

PROBLEM 2: Consider a communication channel that uses ternary signaling with equal symbol probabilities at the source. Let t denote the transmitted signal and r denote the received signal. The following information is known about the channel from measurements.

| $P(r = s_j t = s_i)$ | $t = s_1$ | $t = s_2$ | $t = s_3$ |
|------------------------|-----------|-----------|-----------|
| $r = s_1$ | 0.3 | 0.4 | 0.3 |
| $r = s_2$ | 0.2 | 0.5 | 0.3 |
| $r = s_3$ | 0.5 | 0.1 | 0.4 |

| TABLE 1. | Channel | information |
|----------|---------|-------------|
|----------|---------|-------------|

- (1) Compute the a-posteriori probabilities $P(t = s_i | r = s_j)$ for every pair *i* and *j*.
- (2) How would you choose your decision rule to minimize the probability of error ?

(10 pts.)

(15 pts.)

PROBLEM 3: Let $X(t) = A \cos(\omega t)$ be a random process taking equally likely possible ternary values of ω from the set $\Omega = \{0, \frac{\pi}{2}, \pi\}$.

- (1) Plot the sample functions.
- (2) Obtain the probability distribution and probability mass function of the random variables X(1) and X(2).
- (3) Obtain the conditional probability mass function of X(1) given X(2) = A. Plot the probability distribution function.

PROBLEM 4: Problem 1.4.7 from the text Moon and Stirling.(5 pts.)PROBLEM 5: Problem 1.4.10 from the text Moon and Stirling.(15 pts.)