Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani
Homework #2, Spring 2025

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Jan. 23^{rd}, 2025 **Due date:** Mar. 6^{th}, 2025, 11:59 pm

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PROBLEM 1: Solve problems 4.6.3, 4.7.5, 4.7.6, 4.7.8 from Mark Wilde's book. (12 pts.)

PROBLEM 2: Work out the following problems:

(1) Establish the Schmidt decomposition result when the dimension of the quantum systems A and B are not the same, i.e., in the most general form.

(2) Establish mathematically how Schmidt decomposition can help examine if a pure bipartite state $|\phi\rangle^{AB}$ is an entangled state or a product state.

(8 pts.)

PROBLEM 3: Solve problems 8.3, 8.10, 8.11 from Nielsen and Chuang's book.

(10 pts.)

PROBLEM 4: Consider a qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Suppose phase noise acts on this state, dephasizing the qubit. This action can be described as a unitary action on the qubit such that the rotations $R_z(\theta)$ act on the qubit according to uniform distribution over the random variable θ. Obtain the resulting density matrix. Further, suppose that the longitudinal and tranverse relaxation times of the qubit are T_1 and T_2, respectively. Obtain the final density matrix as a function of all the given parameters, and physically interpret your results geometrically over the Bloch sphere. How do you generalize this setup for a composite system when the qubits are in (a) product state and (b) entangled state? You need to bring in the relaxation time parameters to the composite system carefully within the density matrix formulation. This gives you an idea what happens when the qubits are not identical and what it takes to control relaxation times.

(10 pts.)