Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani
Homework #1, Spring 2025

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Jan. 31^{st}, 2025 **Due date:** Feb. 15^{th}, 2025, 11:59 pm

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PROBLEM 1: Consider the standard quantum teleportation protocol for teleporting a quantum qubit state $|\psi\rangle$. The measurement outcomes in the Bell basis must be relayed to the receiver to reconstruct the quantum state. Suppose the measurement outcome is corrupted by noise which can be modeled using a binary symmetric channel with crossover probabiltiy p, what is the reconstruction fidelity at the output? Suggest a simple scheme to improve this reconstruction fidelity. Justify all your reasoning carefully, including the teleportation part. (6 pts.)

PROBLEM 2: Consider a triangular prism with vertices A, B and C on the top and the corresponding vertices A', B' and C' at the bottom. A spider and an ant are initially sitting on vertices of A and C', respectively. At each time step, both them traverse only along an edge of the prism. The choice of an edge is equally likely from the starting vertex at any time step. At any time while on an edge, they do not reverse their directions. What is the expected number of steps taken before the spider and the ant meet? What is the entropy rate of this random walk process until the spider and the ant meet? (8 pts.)

PROBLEM 3: A rudimentary channel model for reading the charge from M-ary-based flash memory cells can be described using an extended version of the discrete binary symmetric channel extended for the M-ary inputs.

(1) Assuming that the crossover probabilities are the same across the symbols, derive an expression for the channel capacity of the model.

(2) Suppose we have a bad flash memory device due to manufacturing, where the crossover probabilities are *time-varying*. Let $\{Y_i\}_{i=1}^{n}$ be the random variables sensed at the analog-to-digital converter (ADC) output of this $M-$ary cell, corresponding to the inputs $\{X_i\}_{i=1}^{n}$ assumed to be conditionally independent. Let the conditional distribution be given by $p(\mathbf{y}|\mathbf{x}) = \prod_{i=1}^{n} p_i(y_i|x_i)$ over n reads. Let $\mathbf{X} = (X_1, X_2, ..., X_n)$ and $\mathbf{Y} = (Y_1, Y_2, ..., Y_n)$. Obtain $\max_{p(\mathbf{x})} I(\mathbf{X};\mathbf{Y})$ using the setup from the previous part.

(8 pts.)

PROBLEM 4: Answer the following:

(1) Obtain the stationary states of the single qubit Hamiltonian for the following cases: (a) $\hbar\omega Z$ and (b) $\hbar\omega H$, where Z and H are the usual phase flip and Hadamard operators, respectively. Obtain the evolution of the stationary states over time. Physically interpret your results.

(2) Suppose we prepare an ensemble of Bell states $\{\left(\frac{1}{2}, |\Phi^+\rangle\right), \left(\frac{1}{4}, |\Phi^-\rangle\right), \left(\frac{1}{8}, |\Psi^+\rangle\right), \left(\frac{1}{8}, |\Psi^+\rangle\right)\}$ in the lab. What is the expectation of the ensemble over the observables (a) XX and (b) XZ?

(5 pts.)