# Indian Institute of Science

E9: 252 Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam, Spring 2024

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**Name and SR.No:**

**Instructions:**

- This is an open book, open notes exam. You might refer to any papers.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- There is absolutely no collaboration with any human or a bot.
- This is a take home exam. Assigned on March 16th, 2024, 11:59 pm. Turn in by March 23rd, 2024 11:59 pm.
- Do not panic, do not cheat, good luck!

| Question No. | Points scored |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total points | |

PROBLEM 1: A signal $x[n]$ with sampling rate 18 KHz is to be downsampled by a factor $M = 36$. The passband of the signal is from 0-250 Hz. The band extending from 250-500 Hz is the transition band. Pass band and stop band ripples of 1e-3 are desired. Design the best sampling rate conversion system for this specification. Your solution must be the *optimal* one in terms of computational complexity, latencies, along with the filter details etc. Once you setup all the equations carefully, you might write a software code to help you with the computations that you could populate within a table towards distilling an efficient architecture. Justify all the details of your work. (25 pts.)

PROBLEM 2: It is desired to build a filterbank with 3 branches with decimation rates (a) $(3, 3, 3)$ and (b) $(4, 4, 2)$, respectively. If you are permitted to use higher order Nyquist-M filters as a mother filter or a suitable choice of analysis filters, is it possible to arrive at the synthesis filters towards perfect reconstruction? You need to explicity construct the filterbank towards alias cancelation and perfect reconstruction. Show all the details of your work carefully. (25 pts.)

PROBLEM 3: A digital sequence $x[n]$ is emitted in such a way that no two ones appear consecutively. What is the input power spectral density of such a source? It is obvious that $P(0) > P(1)$ for such a source. However, you need to determine these probabilities asymptotically. Suppose this sequence is (a) downsampled by 2, and (b) upsampled by 2. What is the resulting power spectral density of the output sequence in each of the two cases? (25 pts.)

PROBLEM 4: Solve problems 4.29 and 5.26 from P. P. Vaidyanathan's book. (15 pts.)

PROBLEM 5: You are given $N$-linear time invariant (LTI) systems with state space representations $\left(\mathbf{A}_i, \underline{b}_i, \underline{c}_i^T, d_i\right)$ for $1 \leq i \leq N$. What is the overall state space representation if they are (a) connected in parallel, and (b) connected in series? (10 pts.)