Indian Institute of Science

Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani
Home Work #1, Spring 2024

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Jan. 24^{th}, 2024 **Due date:** Feb. 6^{th}, 2024, in class

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PROBLEM 1: Consider all points uniformly and jointly distributed within a sphere in $\mathbb{R}^3$ centered at the origin and radius a units. Obtain the marginal densities and the covariance matrix analytically. Are the random variables statistically independent? Justify. Write a software code to verify your analytical results through alternative means and validate. (10 pts.)

PROBLEM 2: Consider a causal IIR filter $H(z) = \frac{1}{1+\sum_{n=1}^{N} h_n z^{-n}}$. Show that $H(z)$ is BIBO stable if $D(1) > 0$ and $D(-1) > 0$, where $D(z)$ is the denominator of $H(z)$. For the special case of a 2nd order filter, with real h_1 and h_2 prove that $H(z)$ is BIBO stable iff h_1 and h_2 are restricted to lie within a triangular region.

(10 pts.)

PROBLEM 3: Consider the 3rd order IIR filter $H(z) = \frac{1}{1-0.95z^{-1}+2.85z^{-3}}$. The following questions are related to this.

(1) Obtain the state space representation of this system. Using the derived state space representation, write a software code to obtain the response of the system to the input $x[n] = 0.25\cos\left(\frac{\pi}{2}n\right)$, $n \geq 0$. Comment on the filter stability based on the state space representation.
(2) Suppose $x[n]$ is a Bernoulli random process with $P(x[n] = 0) = p$ for all times $n \geq 0$, obtain the output power spectral density.
(3) A digital designer would like to quantize the coefficients of this filter using B bits of precision, which includes the sign bit, a bits for magnitude and the rest for fractional bits. Assuming $B = 4, a = 1$, obtain the filter coefficients after rounding. As a result of this roundoff, what is the variance of the quantization error? Through a pole-zero plot, post quantization, where do the new poles lie w.r.t the original ones?

(15 pts.)

PROBLEM 4: Consider three signals $s_1(t) = u(t) - 2u(t-1) + u(t-2)$, $s_2(t) = u(t) - u(t-1)$ and $s_3(t) = u(t-1) - 2u(t-2) + u(t-3)$ with prior probabilities $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively. Suppose these signals are used for a point-to-point transmission.

(1) Represent the signals through an appropriate signal geometric setup. What is the signal dimension?
(2) Suppose a noisy Gaussian cloud $\mathcal{N} \sim (0, \sigma^2)$ acts independently on the signal coordinates during transmission, derive an expression for the probability of misclassification of the received signals under the MAP rule.

(10 pts.)

PROBLEM 5: An analog is sampled at 20 Kb/s and reconstructed perfectly using an ideal brickwall filter whose digital cutoff frequency is $\frac{\pi}{8}$.

(1) What can you say about the maximum frequency content in the original analog signal?
(2) Write an expression for the reconstructed analog signal from the sampled values.

(5 pts.)

PROBLEM 6 (EXTRA CREDIT): Read the material on superoscillatory signals and write a short note on this significance. Provide your own example of a superoscillatory signal. (5 pts.)