

Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani

Homework #2, Fall 2023

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, $d = \#$ days late

Assigned date: Sep. 24th, 2023

Due date: Oct. 2nd, 2023, 5 pm

PROBLEM 1: Justify if the convex combination of density operators is a valid density operator.

(2 pts.)

PROBLEM 2: Solve the following exercise problems from Nielsen and Chuang's book: Exercises 8.18, 8.19, 8.29, Problems 8.2 and 8.3.

(18 pts.)

PROBLEM 3: In the class we discussed completely positive trace preserving (CPTP) maps. Provide a constructive example of a completely positive trace non-preserving map i.e., CPTNP.

(5 pts.)

PROBLEM 4: A quantum state ρ^A interacts with an environmental state ρ^E unitarily through U^{AE} . Derive the general form of Kraus operators for this channel setup for the quantum state over subsystem A . What are the conditions on U^{AE} if the resulting quantum state is separable? Now, suppose POVM measurements $\{\Lambda_j\}_{j \in \mathcal{A}}$ are done over the subsystem A i.e., post unitary interactions with the environment, what is the resulting quantum ensemble? You may assume any hidden variables required for this setup. State them explicitly as required when you derive the results.

(5 pts.)

PROBLEM 5: Exercise 8.24 from Nielsen and Chuang (extra credit only)

(5 pts.)