Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani
Homework #1, Fall 2023

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Sep. 6^{th}, 2023 **Due date:** Sep. 17^{th}, 2023, 11:59 pm

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PROBLEM 1: Consider a discrete memoryless binary *asymmetric* erasure channel with the following conditional error probabilities i.e., $P(r=1|s=0)=p$, $P(r=0|s=1)=q$ and $P(r=e|s=0)=P(r=e|s=1)=\epsilon$. The discrete random variable realizations r and s stand for receiver and sender sides respectively. The symbol e denotes the erasure in the receiver side. Obtain a closed form expression for the channel capacity. (10 pts.)

PROBLEM 2: Solve the following exercise problems in Mark Wilde's book 2nd edition: 3.3.2, 3.4.4 and 3.5.7. (7 pts.)

PROBLEM 3: In the class we assumed that the ancilla was in the $|0\rangle$ state while proving the no-cloning theorem. Justify if the results of the no-cloning and no-deletion theorem are valid if the ancilla is dependent on the state to be cloned or deleted.

(8 pts.)

PROBLEM 4: In compact disks (CDs), non-binary information can be stored in graded pits by sensing the reflectivities from the pits using a modulation code. Suppose a $(4,1,\infty)$ modulation code is used, where no two non-zero symbols occur side-by-side. Any valid coded sequence on a CD using this code admits concatenations of phrases of the form $\{0^i s\}$, where $i \in Z^+$ with $\cup\{0\}$ and $s \in [4]$ i.e., a string of zeros followed by a non-zero symbol. During the concatenation step, the phrase distribution is assumed to be i.i.d. It is obvious that phrase lengths are non-uniform.

(1) What is the combinatorial entropy of the source that emits such constrained sequences?
(2) Obtain the optimal phrase distribution $p_X(x)$ to achieve this maxentropic limit i.e., $\sup\limits_{p_X(x)} \frac{H(X)}{E(X)}$.
(3) From an appropriate statistical description of the source, empirically show the convergence of the sample entropy to the true entropy. Plot your results. What is the size of the typical set?

(20 pts.)

PROBLEM 5: Prove that strong joint typicality implies marginal typicality for either the sequence x^n or y^n. (Exercise 14.8.1 from M. Wilde's book 2nd edition).

(5 pts.)