Indian Institute of Science

E2: 270 Quantum Information Theory

Instructor: Shayan Srinivasa Garani

MidTerm Exam #2, Spring 2022

Name and SR.No:

Instructions:

- This is an in-class exam for 1.5 hours.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- There is absolutely no collaboration except referring to your crib sheets and homework solutions.
- Assigned on Nov. 22nd, 2022 at 11:30 hrs.
- Do not panic, do not cheat, good luck!

Question No.	Points scored
1	
2	
3	
Total points	

PROBLEM 1: This problem has 2 parts:

(1) Consider a generalization of the quantum hypothesis testing that we did for the binary case. Let $\{(M_i, \rho_i)\}_{i=1}^k$ be k pairs of quantum states and POVM measurements for discriminating the quantum states. Prove that the average correct probability with uniform distribution is upper-bounded by $\frac{d}{k}$, where d is the dimension of the Hilbert space of the operators. Further, show that the average correct probability is upper-bounded by $\frac{d}{k} \max_i ||\rho_i||$ (7 pts.)

(2) In the class, we discussed the strong subadditivity condition. Show that this condition is equivalent to $H(AB|C) \leq H(A|C) + H(B|C)$. (3 pts.)

PROBLEM 2: This problem has 2 parts:

(1) Suppose for any two quantum states ρ^{AB} and σ^{AB}, $||\rho^{AB} - \sigma^{AB}||_1 \leq \epsilon$. Prove that $|I(A;B)_\rho - I(A;B)_\sigma| \leq 6\epsilon \log_2(d_A) + 4H_2(\epsilon)$, where $H_2(.)$ is the usual binary entropy function. You can use any results proved in the class. Any new results must be proved from first principles. (7 pts.)

(2) Consider an ensemble $\{p_X(x), |\psi_x\rangle\}$. Let ρ be the expected density operator of the ensemble. Prove that $H(X) \geq H(\rho)$ i.e., the Shannon entropy is greater than the von Neumann entropy. Under what conditions will the equality hold? (8 pts.)

PROBLEM 3: This problem has 2 parts:

(1) Show that the coherent information of the tensor product channel $\mathcal{N}_1 \otimes \mathcal{N}_2$ satisfies $Q(\mathcal{N}_1 \otimes \mathcal{N}_2) \geq Q(\mathcal{N}_1) + Q(\mathcal{N}_2)$. (7 pts.)

(2) In the class we proved that the Holevo information can be concave or convex depending on whether the signaling states are fixed or the input distribution is fixed. What is the implication of this theorem for optimizing Holevo information over large dimensions? (3 pts.)