Indian Institute of Science

Mathematical Methods and Techniques in Signal Processing

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Home Work #1, Spring 2021

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Apr. 5^{th}, 2021 **Due date:** Apr. 19^{th}, 2021, 11:59 pm.

PROBLEM 1: Consider all points uniformly and jointly distributed within the unit cube in $\mathcal{R}^3$ with bounding axes $0 \leq x, y, z \leq 1$. Obtain the marginal densities and the covariance matrix analytically. Are the random variables statistically independent? Justify. Write a software code to verify your analytical results through alternative means and validate. (15 pts.)

PROBLEM 2: Consider the 3rd order IIR filter $H(z) = \frac{1}{1+1.24z^{-1}+3.67z^{-2}+5.85z^{-3}}$. The following questions are related to this:

(1) Obtain the state space representation of this system. Using the derived state space representation, write a software code to obtain the response of the system to the input $x[n] = 0.5\cos\left(\frac{\pi}{2}n\right)$.
(2) Suppose $x[n]$ described in part (1) is treated as a discrete random sequence, assuming that all the sample values are equally likely, obtain the input power spectral density.
(3) A digital designer would like to quantize the coefficients of this filter using B bits of precision, which includes the sign bit, 3 bits for magnitude and the rest for fractional bits. Assuming $B = 5$, obtain the filter coefficients after rounding. As a result of this roundoff, what is the variance of the quantization error? Through a pole-zero plot, post quantization, where do the new poles lie w.r.t the original ones?

(15 pts.)

PROBLEM 3: Consider three signals $s_1(t) = u(t) - 2u(t-1) + u(t-2)$, $s_2(t) = u(t) - u(t-1)$ and $s_3(t) = u(t-1) - 2u(t-2) + u(t-3)$. Suppose these signals are used for point-to-point transmission.

(1) Represent the signals through an appropriate signal geometric setup. What is the signal dimension?
(2) Suppose a noisy Gaussian cloud $\mathcal{N} \sim (0, \Sigma)$ acts on the signal coordinates during transmission, assuming that the signals are equally likely, derive an expression for the probability of misclassification of the received signals under the MAP rule.

(10 pts.)

PROBLEM 4: An analog signal is sampled at 10Kb/s and reconstructed perfectly using an ideal brickwall filter whose digital cutoff frequency is $\frac{\pi}{8}$.

(1) What can you say about the maximum frequency content in the original analog signal?
(2) Write an expression for the reconstructed analog signal from the sampled values.

(5 pts.)

PROBLEM 5 (EXTRA CREDIT): Read the material on generalized functions in signal processing and write a short note on its significance. (5 pts.)